Investigating Arrow F.O.C

Why does varying the F.O.C of an arrow change the depth at which the arrow penetrates the target?

Physics

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Introduction

Ever since its inception, the bow and arrow's design has stayed nearly the same. Evidence suggests that bows have been around for nearly 60,000 years and became largely popular in hunting almost 13,500 years ago ("History of Archery," 2018). With the main design staying roughly the same for these thousands of years, there was little to no innovation. That is, until recently. Currently, with over 50 different bow manufacturers and nearly 40 arrow and broadhead manufacturers, archery innovation is at its peak. With new compound bows and crossbows coming out every year, as well as new sights, scopes, stabilizers, releases, and risers coming out multiple times a year, the archery landscape is forever improving. These innovations have allowed archers to be able to shoot arrows faster, further, and more reliable than ever before. More recently, there has been a new specific focus on the F.O.C of an arrow and seeing what happens to the arrow as this value changes.

The F.O.C of an arrow, or front of center, is the percentage of the arrow's weight that is in the front half of the arrow (Myers, 2019). This means that the higher the F.O.C is, the more weight there is in the front half of the arrow and the more top heavy it is. F.O.C is one of the main parts responsible for the arrow's trajectory as well as the arrow's impact on the target. It is generally well accepted by the archery community that an arrow's F.O.C should range between 11 and 13 percent for the everyday archer ("What is F.O.C," 2019). Falling at the bottom of this agreed-upon range, most Olympic archers prefer to have their arrows with a value near 11 percent and for some, even lower. This is because the arrows with a lower F.O.C are supposed to have a more stable, repeatable flight for the long ranges that Olympic archers are required to shoot ("Fita Arrow FOC," n.d.). Whereas most hunters prefer a range higher than that of the recommended, somewhere around 15 percent, because it should result in a deeper arrow penetration (Reaser, 2019). However, despite these beliefs and practices, there are few clear definitions of what the difference in this range will entail for the arrow or what will happen outside of this range available. More specifically, there is limited to no data or trends to back up the claim that a higher F.O.C will always cause a higher penetration into targets.

Although, there are a few tests online that do attempt to show the differences in this range. However, they cannot be trusted. Throughout many of these tests and trials, the experimenter attempts to study the effects of a changing F.O.C by changing the weight of the arrow tip. While this does change the F.O.C of the arrow, it also changes the overall mass of the system. This means they are changing and measuring the effects of multiple variables at once, the mass of the arrow tip and the F.O.C. So, it is unclear which aspect is causing the change in the penetration depths, the F.O.C, or the mass of the arrow tip.

This essay is an attempt to study how and why differing the F.O.C value causes a change to the penetration of an arrow. The objective is to see if differing the F.O.C will change the velocity of the arrow. A change in the arrow's velocity would cause a shift in the momentum of the arrow, as well as the kinetic energy of the arrow. Which in turn, could explain the difference in the penetration.

Background information

The first step in calculating the F.O.C of an arrow is to divide the arrow's overall length by 2. Then, measure from the arrow's balance point to its nock. After that, subtract the center of the arrow measurement, the overall length divided by 2, from the balance point. From there multiply by 100 and then divide by the arrow's overall length. The number at the end is the

F.O.C of the arrow. Or, more simply put, the F.O.C value = $\frac{100(A-\frac{L}{2})}{L}$ where A is the distance from the balance point to the nock and L is the length of the arrow (Myers, 2019).

Throughout the experiment, all arrows will be shot through a chronograph on their way to the target. A chronograph is a tool that can measure the velocity of almost any projectile. The one used in this experiment was a Caldwell G2 Ballistic Precision Chronograph. The chronograph also came pre-calibrated within +/- 0.25% of the true velocity and could measure anywhere from 5 to 9,999 fps ("Ballistic Precision G2," 2019).

Another tool used throughout the experiment was the Hooter Shooter. Which is a machine that a bow gets locked into, which is used to shoot the bow with perfect form. It perfectly emulates how a person would shoot, while getting rid of the variation, inconsistencies, and human error that would normally come with it. The Hooter Shooter also uses a winch to draw the bowstring back. This allows the user to draw the bow back to the same distance after every shot. By having a bow locked into a fixed position and having to turn a winch to pull the bowstring back, the bow can shoot the same every single time. In fact, the Hooter Shooter is so exact that it can shoot the same arrow into the same spot shot after shot ("Spot-Hogg Hooter Shooter," n.d.). Meaning that any changes seen in the arrow's velocity and penetration will be a direct result of the changing F.O.C in the arrow and not because of human inconsistencies.

In order to try and see why changing the F.O.C changes the penetration of an arrow, momentum and kinetic energy were used. Momentum is basically inertia in motion. Meaning that the more momentum an object has, the harder it would be to try and stop that object ("Momentum", n.d.). This would explain why certain arrows can penetrate further into the target. Momentum is equal to mass times velocity, or $\rho = m \times v$("Momentum," n.d.). So, an object with a lot of mass and/or velocity would, in turn, also have a lot of momentum. Similarly, kinetic energy is the amount of energy that the arrow has while moving from the bow to the target. The equation for kinetic energy is $E_k = \frac{1}{2}m \times v^2$("Work, Energy, and Power," n.d.). Meaning that an object with a greater amount of velocity will have a greater amount of kinetic energy. So, since energy is commonly described as the ability to do work, it could explain why an arrow with more velocity would penetrate further into a target. In both cases, mass is staying the same throughout the system, so any change in the velocity of the arrow will be the sole cause of the change in momentum and kinetic energy. This would then explain the changing impact depths seen at varying F.O.C percentages.

Experiment Setup

To change the F.O.C of the arrows without changing the overall mass of the arrows, weighted BBs were placed throughout the inside of the arrow. Since arrows have a hollow center in order to cut down on their overall mass, it was easy to get inside and place them. To do this, the nocks of all the arrows were taken off and then 6 BBs all weighing 0.49-grams were lodged through the inside of the arrow. Each arrow had 6 BBs placed throughout different areas of the arrow's shaft, allowing for an easy change to the F.O.C while still maintaining the overall mass constant. The BBs were held inside the arrow due to friction. Since the BBs could barely fit through the arrow, they were not able to move unless there was a substantial outside force. The further up the BBs were in the arrow, the higher the F.O.C was. This was because there was more weight the further up the arrow. Then, all of the new modified arrows used had a range of differing F.O.C values from 9-15 percent. This way data could be gathered for both of the two professionally preferred F.O.C values, as well as some of the values surrounding them. The arrows used throughout the experiment were Beman Carbon Defender 500s. Each arrow was

72.5 centimeters long with a 6.420-gram arrowhead, and all weighing in at a total of 23.389-grams before modifications, and 26.33-grams afterward.

Before the experiment started the Hooter Shooter was set up and tested. This was to make sure that any changes measured were solely from the differing F.O.C. In order to do this, a few different arrows were shot multiple times and then measured. The measurements showed that the velocity and penetration remained the same after multiple shots with different arrows. This meant that throughout the remainder of the experiment, each arrow had to only be shot once, as any more would be redundant.

Also, before the experiment started, the chronograph and target needed to be set up. For the experiment, the chronograph was placed in between the target and the bow, around the halfway point. Due to the low lighting conditions on the range, additional LED lights were used on the chronograph for increased accuracy and repeatability with the readings. For the target, a brand-new stiff plastic foam target was used. This was to ensure that there were no pre-existing weak spots in the target that could allow for easier arrow penetration.

While testing the chronograph and the hooter shooter safety measures were taken into account. There were clear instructions on how to use both machines and someone to help nearby. While using the hooter shooter, someone with more experience would help set it up and fire it once everyone was out of the way and behind the contraption, as to avoid any serious injuries that could occur. These precautions were also carried out throughout the remainder of the experiment to ensure the safety of everyone involved.

- Length of all arrows:

$L_a = (72.5 \pm 0.05)$ cm

- Mass of individual BBs

$M_b = (0.49 \pm 0.005)$ g

- Mass of arrows without added BBs:

$M_a = (23.389 \pm 0.005)$ g

- Mass of arrows with added BBs:

$M_a = (26.33 \pm 0.05)$ g

The errors for all the measured values are half of the smallest unit. However, in the case of the arrows with the added BBs, a bigger error margin was used to account for the larger error margin of the BBs' mass, rather than that of the arrows.

Experiment Procedure

The idea of the experiment was to test how varying the F.O.C of arrows impacts the arrow's penetration depth in a target. Then, from there, the goal was to see if there was a correlation between the penetration and the momentum observed in the varying arrows. The bow used throughout the experiment was a BowTech Fuel with a 45-pound drawback and a 47 cm drawback length. All of the arrows that were shot were fired from the same bow, which was 5 yards away from the target. Before each arrow was fired, their F.O.C was remeasured again to ensure that no changes somehow occurred by traveling. Each arrow was then shot through the chronograph before hitting the target. For each shot, the bow was locked in place on the Hooter Shooter, in order to guarantee consistency. After impacting the target, the length the arrow traveled into the target was measured, as well as the velocity of the arrow as it passed through the chronograph. Once taken out of the target, the F.O.C was remeasured again in case a reshot was required due to the chronograph not picking up the arrow's velocity. The entirety of the experiment was done inside an indoor archery range. This was to make sure that there were no other extraneous variables during the experiment that could occur if done outside, like the wind for example.

Experiment Results

Arrow F.O.C % (± 0.05)	Arrow Depth in Target (cm) (± 0.05)
9.17	29.64
10.20	30.59
11.17	32.18
11.93	34.72
13.18	36.62
14.01	37.35
15.05	38.05

Table 1: Shows the data raw data collected from the experiment.

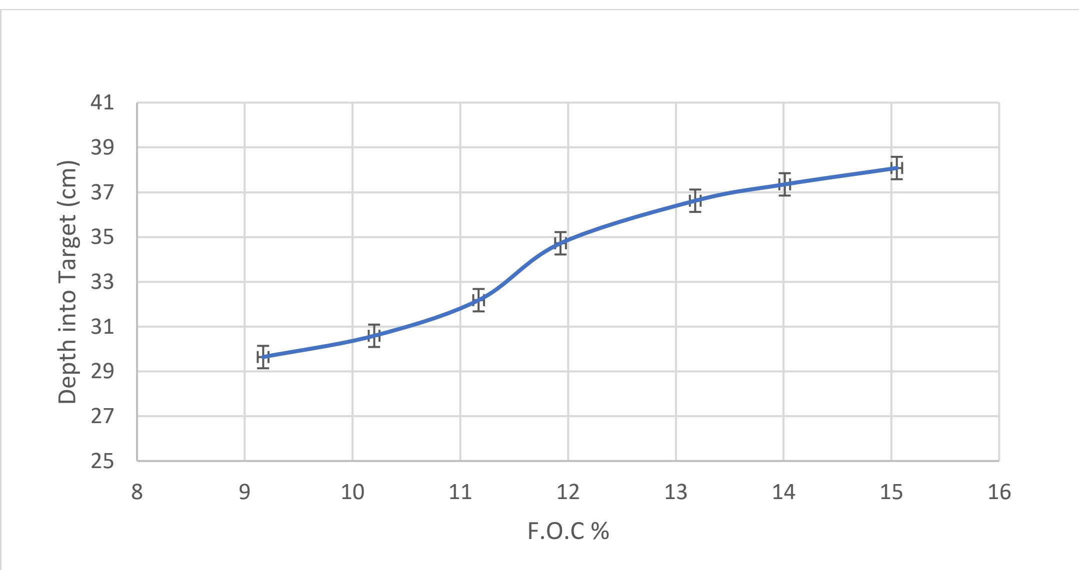


Figure 1: Depicts the relationship between F.O.C % and the depth the arrow traveled into the target.

As depicted in the table above, and seen in the graph above, there is a positive trend between the F.O.C of an arrow and the penetration into the target. Meaning that as the F.O.C percent increases, so will the depth that the arrow travels into the target. Although, it is interesting that this relationship between F.O.C and penetration depth is non-linear. Based on articles online, one would assume that F.O.C and penetration depth are directly proportional. But this seems to not be the case. The graph illustrates that as the F.O.C percent begins increasing, the depth into the target increases slowly. Then as the F.O.C approaches 11 percent, the slope of the graph gets greater until finally beginning to taper off around 13 percent. By the looks of the graph, asymptotes seem to appear around 29 and 39 cm into the target. It would be interesting to have continued testing different values of F.O.C percentages to see if the trend would have continued and if the asymptotes would have appeared. However, this would have been more difficult with each increasing or decreasing value. As one moves more mass around the arrow, there still are other parts of the arrow. This means that it is impossible to get certain values of F.O.C, as there will always be mass in the back half of the arrow. This leads to one of the limitations of this graph, as it is impossible to have all the weight in the front of the arrow and none of the weight in the back, and vice versa. In fact, taking a closer look at the equation for arrow F.O.C, F.O.C% = $\frac{100(A-\frac{L}{2})}{L}$, it is possible to find the theoretical minimum and maximum F.O.C percent. After substituting the length of the arrow, in this case, 72.5 centimeters, in for L and solving for A, the distance from the nock to the balance point, these values are apparent. Using the current arrow setup and equation, the maximum F.O.C of an arrow is only 49 percent and the minimum is 1 percent. It is because of this that the relationship is cubic, with extremes on both ends of possible F.O.C.

Arrow F.O.C % (± 0.05)	Velocity (m/s) (± 0.5)
9.17	61.3
10.20	61.5
11.17	61.9
11.93	62.7
13.18	63.3
14.01	63.7
15.05	64.0

Table 2: Depicts the other raw data collected in the experiment, velocity.

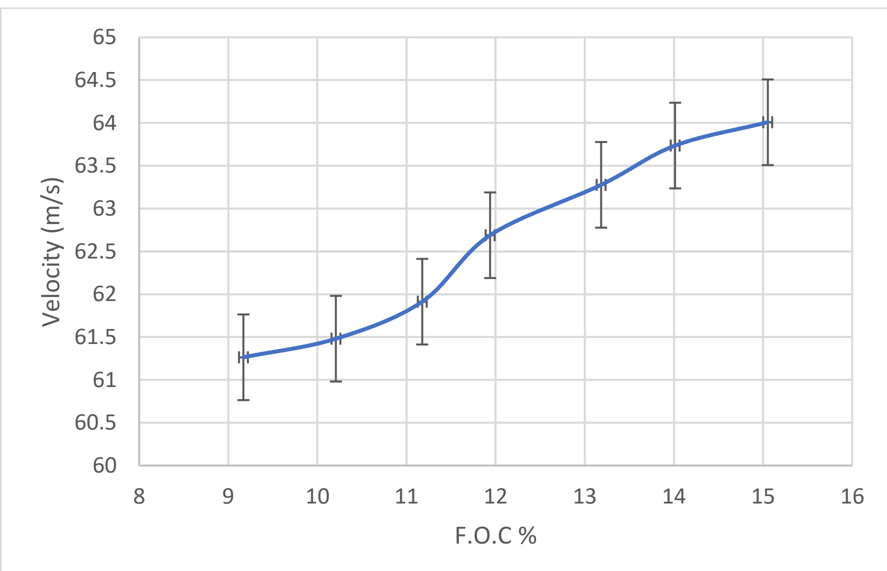
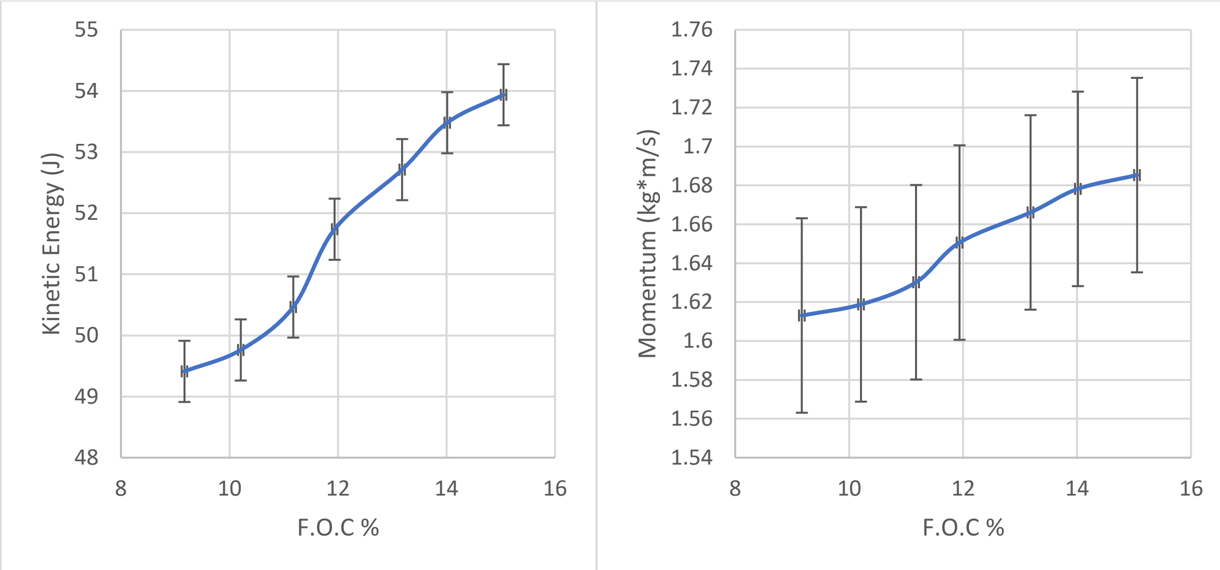


Figure 2: Illustrates the relationship between F.O.C and velocity.

The diagrams above, table 3 and figure 3, show the relationship between an arrow's F.O.C and its velocity. As discussed with the previous figure and table (table 1 and figure 1), there is a positive relationship between arrow F.O.C and impact depth. This can be explained by the positive trend apparent in the F.O.C vs velocity graph, figure 2. As the velocity of an arrow increases, both the momentum and kinetic energy of said arrow will increase. This would explain the changing impact depths. Also similar to the slope in figure 1, the slope seen in the graph above seems to slowly increase, then hit a peak and start to decrease. On the graph at these points, there appear to be asymptotes. Specifically, the asymptotes are apparent around 61 and 64 m/s. These asymptotes suggest that changing the F.O.C can only change the velocity of an arrow so much, and eventually any further changes to the F.O.C won't result in any significant change. This was also something that was previously seen in table 1 and figure 2. More simply, the F.O.C only has so much effect on the velocity of an arrow. So, the measurable impact that it has on velocity should lessen as it approaches both extremes.

Arrow F.O.C % (± 0.05)	Kinetic Energy (J) (± 0.05)	Momentum (kg*m/s) (± 0.05)
9.17	49.41	1.60
10.20	49.76	1.62
11.17	50.46	1.63
11.93	51.74	1.65
13.18	52.71	1.66
14.01	53.48	1.67
15.05	53.94	1.68

Table 3: Shows the data collected between arrow F.O.C remeasured in the form of kinetic energy and momentum.



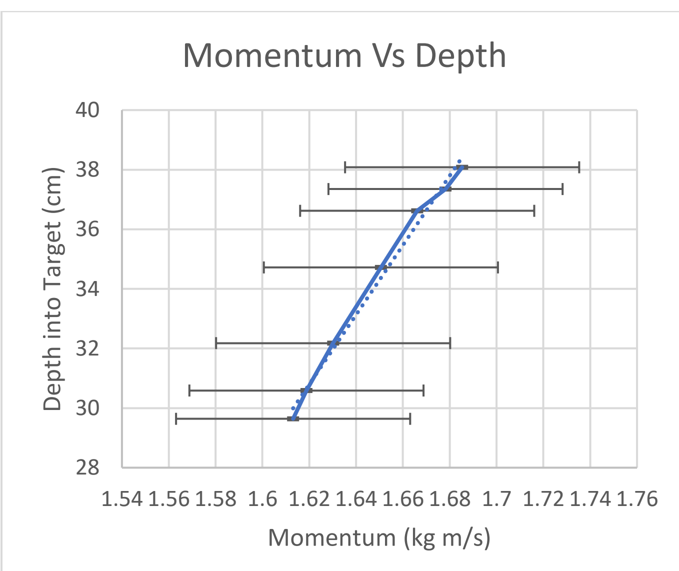
Figures 3 and 4(respectively): Illustrates the relationship between F.O.C and kinetic energy and momentum.

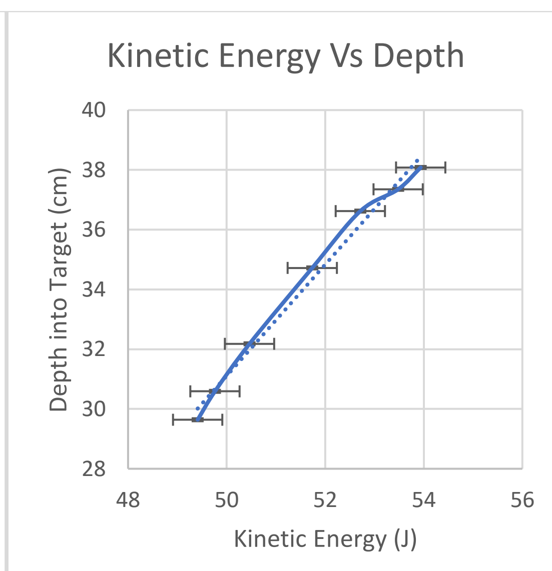
Unlike table 2 and figure 2, which measured the F.O.C versus velocity, the diagrams above, table 3 and figures 3 and 4, measure F.O.C versus momentum and kinetic energy. Meaning, that now there is the mass of the arrow included in the data as well. Similar to how the F.O.C percentage could only change the velocity so much before any more changes resulted in no gain or loss, the F.O.C will only be able to change the momentum a certain amount. Which appears to range between 1.61 kg*m/s and 1.69 kg*m/s in figure 7, which are consequently the asymptotes on the graph as well. This can also be seen in figure 6, where the kinetic energy seems to slow down and never pass 49 and 54 joules. This makes sense, as the only difference between the variables now is the inclusion of mass, and how impactful velocity is. Although both graphs look the same, they are entirely different. Figure 4 is measuring momentum, or the product of mass and velocity. Whereas figure 3 is measuring kinetic energy, which is the product of half the mass and the velocity squared. This means that if velocity should double, the kinetic energy that the arrow has will increase by a factor of 4. Should the velocity triple in quantity, then the kinetic energy will increase by a factor of 9, and so on. This is vastly different compared to momentum, were if velocity where to double, then so would the momentum. However, both of these graphs could explain how changing the F.O.C could change the impact depth. If one of these relationships is a direct cause, then as the F.O.C increases so would the velocity, which would cause one of these quantities to increase as well, explaining the different impact depths seen by different F.O.C values.

Momentum (kg*m/s) (± 0.05)	Arrow Depth in Target (cm) (± 0.05)
1.60	29.64
1.62	30.59
1.63	32.18
1.65	34.72
1.66	36.62
1.67	37.35
1.68	38.05

Kinetic Energy (J) (± 0.5)	Arrow Depth in Target (cm) (± 0.05)
49.41	29.64
49.76	30.59
50.46	32.18
51.74	34.72
52.71	36.62
53.48	37.35
53.94	38.05

Tables 4 and 5(respectively): Shows data for momentum and kinetic energy, and how that related to arrow depth.





Figures 5 and 6(respectively): Shows a direct comparison and arrow's momentum and kinetic energy vs their depth.

Finally, in tables 4 and 5, as well as figures 5 and 6, there are more positive correlations. However, for the first time, they appear to be linear. This linear correlation between kinetic energy and impact depth suggests that as the kinetic energy of the arrow increases, the depth at which the arrow travels into the target will increase at a set amount as well. This seems to make sense, if an arrow has more energy, it should be going further into the target. However, this may only appear linear due to the limited domain range. If the momentum or kinetic energy would continue to decrease, then eventually the depth into the target would become negative, which does not make any sense in the real world. Even if the arrow travels the opposite way of the target, the depth would remain zero rather than going into the negatives. These relationships are likely very similar to the ones discussed above, where asymptotes begin to appear at the relative extremes. It is only not apparent in the graphs because the domain is limited.

Although both momentum and kinetic energy seem to explain why the arrow travels deeper into the target. Only momentum accurately relates the two. This is because in archery once the arrow hits the target its kinetic energy gets dissipated and travels throughout the target, rather than all being focused in one general area (Freel, 2016). This kinetic energy also gets transferred into heat and sound energy and is not simply the energy focused in one small area that accounts for the impact depth. It is also the measurement of the energy the arrow has rather than the force it carries. However, momentum does explain the relationship better. Since momentum is the measurement of the force that the arrow is flying with, it also reveals the force the is required to stop it. As it would be the same but in the opposite direction. As the momentum increases, so does the force required to stop an arrow. However, since the target is staying the same from shot to shot, an arrow with more momentum will go further into the target before being stopped as it has more force. Similarly, as the arrow gains more momentum, the depth that that arrow penetrates the target will increases at a fixed amount as well. Which does make a lot of sense. Since momentum, as was previously described, as simply inertia with motion, the more inertia the harder it is to stop. So, because the momentum is increasing, the arrow will carry more inertia, which makes it harder for the target to stop it, thus allowing for the arrow to continue going deeper into the target.

However, this is where things start to get confusing. Currently, there are no papers explaining why F.O.C causes a change in velocity. When previously studied, experimenters also changed the mass, which does explain the changing velocity. But in this case, the mass is constant throughout the experiment. One hypothesis that could explain this has to do with the wave pattern of the arrow after being shot. Once an arrow is shot, most of the force coming from the string gets transferred into the arrow. But the arrow can't take it all upfront at once. The back of the arrow is impacted first and then the force travels up the arrow to the front. This is what causes the wave-like bend in the arrow after it is shot. The point of balance in the arrow, which is impacted by the F.O.C, and the nock both act as the nodes for the wave in the arrow. And since the higher the F.O.C, the further up the balance point is, the bigger the wave in the arrow. This could explain why higher F.O.C values relate to more velocity. If the wave had a bigger amplitude, then once that arrow snapped back to its original, upstretched shape, it could have been given more speed. However, this is simply a hypothesis. More testing should be done as well as recordings with a high-speed camera to confirm or disprove this theory.

Conclusion

Now, for the first time, there is data showing the difference between varying F.O.C values and their impact depths. As well as data attempting to explain why. Clearly, a positive trend is apparent throughout all graphs and data tables during the experiment. Meaning that as one increases the F.O.C of an arrow, the arrow penetrates further into the target, seen in figures 1 and 2. Thus confirming the idea that as one increases the F.O.C the impact depth will increase as well. This was explained by figures 3 and 4, which illustrated that as F.O.C increases, so does the velocity of that arrow. And, since there was a positive trend between F.O.C and velocity, there was also a positive trend between F.O.C and momentum and kinetic energy. This meant that both momentum and kinetic energy are positively correlated to an arrow's impact depth.

Though, only momentum can be used to explain why F.O.C changes the impact depth. Since momentum is inertia with motion, it is a force quantity. Meaning that the more momentum that an object has the harder it is to stop. So, an arrow with a higher F.O.C which in turn gives it a higher velocity, will have more momentum. The more momentum that an arrow has, the harder it will be to stop. This is why on the arrows with more momentum, there is also a greater impact depth length. Whereas kinetic energy is simply the energy that the arrow has in flight. Meaning, that unlike momentum, kinetic energy is an energy quantity. So, when the arrow strikes the target, all that energy is transferred and dissipated rather than being used to go through the target.

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