Examiner rated this a B

The Geometry of Time and Seasons

How does geometry applied in astronomy determine the measurement of time and explain how seasons work on Earth?

Mathematics Extended Essay

Word Count: 3,890

Table of Contents

Introduction

As a child, the studies of the science and math of the celestial universe was personally intriguing. Combined with an early appreciation for mathematical functions and exploration, the way in which the natural world and universe functioned was a topic that had infinite, intriguing explorations. A topic of applying the geometric branch of mathematics to astronomy in terms of the space and movements between celestial bodies could introduce me to some of the basics of astronomy, a branch of science that I personally consider to be the one of the most adventurous. This topic developed into focusing on the Earth's movement on its orbit around the sun, and how the sun's light and radiation, over the course of each day and year, created natural occurrences on the planet, and how mankind used methodology from geometry to fundamentally understand and maintain a system of benefiting off the knowledge of these occurrences. What mathematical models and geometric principles were used to initiate these systems, as well as continually keep track of them?

The following composition intends to focus on the geometry of the earth in relation to the sun, and the way in which that governs both the duration of the day and year and the climate of Earth. Moreover, **how does geometry applied in astronomy determine the measurement of time and explain how seasons work on Earth?** Historical methodologies that led to modern methodology of the time and seasons systems will initially be analyzed. This begins with pre-scientific beliefs of the day, night, and seasons, followed by the sundials being used to determine the passage of time in ancient history. The geometry that went into the functioning of sundials provided a basis for the modern day time system used in clocks and calendars, based on more accurate, technology-based studies of the Earth's rotation and revolution around the sun. There will often be a focus on spherical geometry, as methodology of this subject helped create time zones to keep track of time better. The trigonometry of the span between the Earth and the Sun also allowed society to understand seasonal changes throughout the year, and the cause of differing climates across the globe.

Understanding all of this is immensely valuable in having knowledge of the basics of the natural phenomena that occurs on our planet and analyzing further progressions throughout time. These studies are beneficial in evaluating how we can use this for an even greater understanding in the future. It is also greatly fascinating to have a better appreciation and admiration for the beauty and usefulness of mathematical models in creating deductions from scientific studies.

The geometry that occurs between the Earth's changing position in relation to the sun is a fundamental application to astronomy, because it is necessary to create a reliable system of time and understanding seasonal changes.

History and Sundials

History of Time Measurement and Seasons Study

Before mathematical and scientific practices arose, religious and mythological beliefs were applied in response to the observation of day, night, and seasons. Various ancient philosophies were important in the creation of terminology and the roots of mankind's modern day time structure. In ancient Greek mythology the story of Demeter, the goddess of harvest, is about a mother who is happy during summer and spring where she can see her daughter, and depressed during fall and winter periods when her daughter is in the underworld with Hades. (How the Four Seasons Came to Be in Greek Mythology, n.d.) Though it is mythology, the idea of spring and summer being a time of warmth and growth of crops, and fall and winter being otherwise, creates the basis of our seasonal cycle. The ancient Greeks also believed day and night occurred because the sun rode across the sky in a chariot. (Dr. Whitlock, n.d.) Though they originally identified celestial objects as gods and spirits, these beliefs still gave roots to current astronomy.

It is widely known from scientific history that there was originally the world-wide belief that the Earth had a flat surface. This was disproven in around 200 BCE by Greek astronomers that measured the shadow from the Sun on a stick in two separate locations. (Finio, 2017) During the same time of day, the stick in one area created a shadow that was at a 7° angle from the stick, while the shadow in a farther area came down almost directly at a 0° angle. This revealed that the Sun's light came down at different angles in different locations, proving there was a curve on the Earth's surface. Thus, the Earth was proven to be a sphere. The same ancient Greek mathematician, Eratosthenes, also successfully calculated the Earth's circumference, which will be discussed later. The idea of the Sun's light coming down at an angle will also be important later on.

Sundials Development and Usage

Mathematical practices from ancient Egyptian and Greek societies led to the development of sundials with trigonometric functions. Sundials are instruments that are used to measure the passage of day using shadows casted on the device. The trigonometric and arithmetic mathematics involved in using the sundial initiated us dividing up the day in separate units. (Vincent) Their usage was for purposes of survival, as it was fundamental to determine how close nightfall was. This has stuck with us as a key societal and survival basic: having reliable time to know how long is left in the day. The sundial's most important component is the gnomon. The gnomon is the standing part of the sundial that is designed to cast the shadow onto the ground. (Toothman, n.d.) Due to the Earth's axial tilt and revolution around the Sun, the position of the Sun in the sky changes after many days, and the gnomon must reflect this. (How do sundials work?, 2008) Though there are many variations of sundials, the most commonly found is the horizontal sundial, which sits flatly on the ground. (8 Different Types of Sundials, n.d.) Depending on its location amongst the globe, the gnomon should be parallel to the axial tilt of Earth, and the tilt of a gnomon of a given location depends on the latitude of said location.

Figure 1 Sundial (stick is the gnomon)

(HORA Horizontal Sundial, n.d.)

The purpose of this is so time can be accurately measured for any day of the year, since the Sun's angle of declination changes throughout the year depending on the Earth's location on its orbit. To make the sundial parallel to the direction of the poles of the Earth, ancient Greek astronomers used the Sun and the stars in the horizon to measure their latitude. (Latitude and Longitude, 2019) Using the angle of elevation of the Sun in the sky, the time of day can be determined on a sundial. (PV Education, 2019) This angle of elevation is called the hour angles, which is the angle the Sun is at from the ground, revealing the time of day. Looking at the shadow formed by the gnomon of the sundial, a tangent function can be applied to determine what time of day is represented by each angle. *H* is the hour angle line on the sundial in relation to the noon line, the *T* angle represents the hour angle from the sun in terms of height , and the *L* is the latitude: (Vincent)

$$H = \tan^{-1}(\tan T \sin L)$$

Detroit at 4 pm on an August day will be used as an example:

$$H = \tan^{-1}(\tan 60° \sin 42.33°)$$

60° for *T* because that is the angle of the shadow at 4 pm, and 42.33° is the latitude of Detroit.

$$H = \tan^{-1}(1.732 \times 0.673)$$

$$H = \tan^{-1}(1.166362479)$$

$$H = 49.39132269$$

The hour angle is 49.39°.

Tangent functions are so commonly used for determining hour angles because the gnomon represents a length opposite to the angle of inclination of the Sun's ray, and the surface of the dial is adjacent to the actual hour angle that is formed from the Sun's rays. Tangent is used take the side opposite from theta angle and divide it by the side adjacent to theta. When looking at a simple vertically standing stick that is perpendicular to the ground, a simpler model is used. *H* is once again the hour angle, *S* is the ground surface, and *O* is the height of the object that creates the angle:

$$\tan H = \frac{S}{O}$$

As an example, a flagpole with the height of 9 feet creates a shadow that reaches out to 6 feet away from the pole:

$$\tan H = \frac{9}{6}$$

$$\tan H = \frac{3}{2}$$

$$H = \tan^{-1} 1.5$$

$$H = 56.30993247$$

The hour angle is 56.3°. If the shadow is stretched towards the east, this means the Sun is in the west. It is commonly known that the Sun shines directly down at 12:00 pm. This happens because it is solar noon, when the Sun's rays are perpendicular to a given line of longitude. (Nilsson)

Since 0° is the hour angle at zoon, and the 90° is at sunset, assuming sunset was at about 9 pm, the time is around 5:38 pm, according to this angle.

The sundial was the first of countless methods of keeping track of time using geometric methods. The importance of the sundial's time measurement led to how time is modernly divided and broken down today, specifically the hour and minute system used on the modern clock. It also gave roots to astronomical studies of the changes of the Sun's position throughout the day and year.

Time Division and Time Zones

Sundial to Clocks

Various factors such as discoveries from the sundial, astronomy methodology of the galaxy, and geometry methodology of the earth's rotation and revolution led to modern society having time divided up the way it is throughout the day and year. The sundials have a parallel to the globe, specifically in terms of time zones, that leant to how the day is divided up and having distinct time zones. Using knowledge gained from sundials about the angle of inclination throughout the day and the angle of declination of the Sun throughout the year helped create a through, accurate calendar system.

How did today's system of time passage come about? Geometry and general mathematics are applied in this realm of astronomy, because of the rotation and revolution of the Earth around the sun, the structure of the Earth as a sphere, and the Earth in relation to other celestial bodies. (Heilbron, 2020) Observations of celestial bodies in ancient astronomy practices also paved way to some guidelines of geometry. A specific deduction is the rule and fact that all circular objects are 360 degrees. After the observation that the Earth completes about 365 rotations once a revolution around the sun is completed, along with the ancient Babylonians using 60 as a base number, 360° worked evenly to represent the number of degrees in all planets. (Marshall, 2015) 60 and 360 also work very well as being readily dividable into other numbers, since both contain many useful factors. The ancient Babylonians originally used 10 hours on their sundials, plus one for the beginning of the day and one for the end. Hence, roughly 12 hours of day and 12 for night. (Why are there 24 hours in a day?, 2011) They are credited with a vast effect on the subdivision of hours and minutes and with circles containing 360 degrees. (Why are there 24 hours in a day?, 2011)

The Earth's Movement Around the Sun

Physics and geometry methodology were used to study the Earth's rotation and revolution in order to develop our system of time passage. The Earth rotates at a velocity of about 1,037 miles every hour. (Herman, 1998) For the circumference of the Earth, Greek astronomer Eratosthenes, in 200 BCE, found the distance of two different locations that were 7° apart on the globe. Knowing that was 7 out of 360 degrees of the Earth, he used trigonometry to find the total circumference. His results were close to the true, modernly proven circumference: 24,850 miles. (Finio, Measure Earth's Circumference with a Shadow, 2017) How long does it take for the Earth's entire circumference to be rotated 360 degrees to its original position? Using some division:

$$24{,}850 \div 1{,}037 \approx 23.96$$

This calculated amount of almost 24 hours is what is known as a full day, and about 365 days make up a year. It is slightly less than 24 hours, hence the previously explained leap day every 4 years. The number of hours in a day rooted from sundials, since they were in sync with the later developed time system.

Using Parallax

The time length of the Earth's revolution was roughly calculated over many centuries by observing seasonal changes and was more accurately developed using trigonometry. How was the Earth's revolution length determined using geometry to figure out the distance between different celestial bodies, though? Astronomers had to figure out complex, creative ways to quantify this distance. The Earth's distance from the Sun was calculated using a method called parallax. Parallax is a term for measuring the distance of object *B* from object *A* from the point of view of object A at two different locations. Using two different locations at different times is used to determine the true distance of object *A* to object *B*. In this astronomical context, object *A* is the Earth on its orbit during winter and summer, and object *B* is the sun. (Lucas, 2018) This concept is how astronomers find the distance of the Earth in relation to other celestial bodies, ranging from planets in the solar system to extremely far away stars. (Tourmen, 2020) Trigonometric parallax uses a triangle between the Earth, another celestial body, and another triangle connecting the celestial body to stars far away in the background.

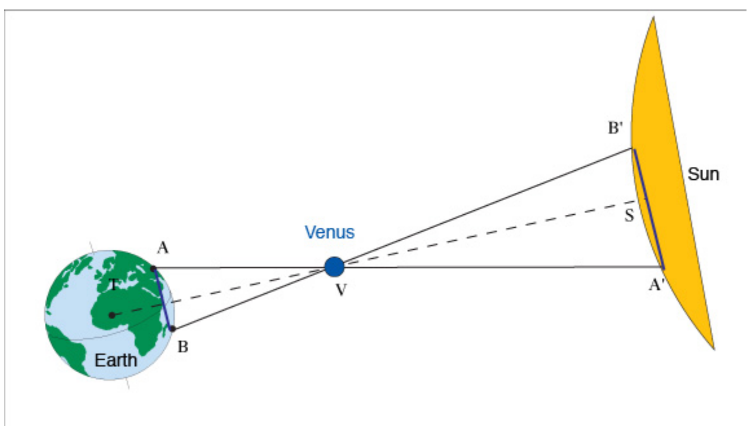

Figure 2 Parallax of the Earth and the Sun using Venus

(The Venus Transit 2004, n.d.)

If the distance of the Earth to Venus is obtained by using the parallax from two different locations of on the globe, the distance from the Sun can be obtained as well. A method used in the 18th century allowed astronomers to find the angular positions of Venus at two different locations on the Earth during a transit, and finding the parallax for the distance of the sun. (Rehnberg, 2015) Whilst having multiple people at different locations on the Earth during a transit, the follow equation can be used: (Finding the AU: How the Transit of Venus Tells Us Our Distance from the Sun, 2012)

v= Venus, *a* and *b* = separate observation points on the Earth, *d*= distance of Earth from Venus

$$\tan\frac{v}{2} = \frac{(0.5(a-b))}{d}$$

Manipulating this equation to solve for the distance of the Earth from Venus:

$$2\times tan\frac{v}{2} = \frac{(0.5(a-b))}{d}\times 2$$

$$\tan v = \frac{(a-b)}{d}$$

$$d\times\tan v = \frac{(a-b)}{d}\times d$$

$$d\tan v = (a-b)$$

$$d\tan v/\tan v = (a-b)/\tan v$$

$$d = \frac{(a-b)}{\tan v}$$

Physics laws say that the distance of the Earth to Venus is 28% of the Earth to Sun distance (Finding the AU: How the Transit of Venus Tells Us Our Distance from the Sun, 2012), so:

Now, *d* = distance of the Earth from the Sun

$$d = \frac{260{,}975{,}590}{0.28} =$$

d = 92.85714286≅

d = 93 million miles

24/7 365 Days a Year

With the Earth having a 24 hour period of rotating on its axis as it revolves around the Sun, time zones must exist for the time of day of different longitudes on the Earth. The Earth is a sphere with 360°. There is a recorded total of 24 hours in each rotation on its axis. To keep a specific enough time for each region of the earth in an organized, non-convoluted way, each time zone is kept only an hour apart. There are 24 time zones, so:

$$360 \div 24 = 15$$

Hence, there are 15° of longitude per each time zone. Longitude, not latitude, because the time zones run from north pole to the south pole, so from the top to the bottom of the globe. (Gedge, 2013) As a deduction from this math, the Earth moves 15° every hour, so by 24 hours it completes a 360° rotation.

Based on globally agreed measurements, the average distance of the Earth from the Sun is 93 million miles (average because the Earth's orbit is a slight oval and not perfect circle). (Sharp, 2017) This distance is commonly known as the *Astronomical unit (AU)*. (au (Astronomical Unit), n.d.) Using this distance as the radius, the length of the circumference of the Earth's orbit can be calculated.

Circumference Equation: $C = 2\pi r$

$$C = 2\pi(93{,}000{,}000)$$

$$C = 6.283 \times 93{,}000{,}000$$

$$C = 584{,}336{,}233$$

This calculated distance of about 584 million miles is the length for the Earth to fully orbit the Sun. Since there are points in the year when the Earth is closer to the Sun and others when it is farther, the circumference will be estimated to 584.3 million miles. The speed of the revolution of the Earth on its orbit is about 66,627 miles per hour, and that is 1.6 million miles every day. (Howell, 2018) Again, taking the circumference of the Earth's orbit and dividing by the miles per day:

$$584{,}300{,}000 \div 1{,}599{,}048 = 365.404$$

This is close to the actual, reported length of the Earth's revolution: 365.25 days. The distance from the Earth on its orbit to the Sun is an important backbone to our time and calendar system, as it assists in the quantity of each second, minute, hour, day, month, and year.

Of course, since a day is actually 23 hours, 56 minutes, and a few seconds, leaving about a quarter of a day over 365 days, a leap day occurs every 4 years. (Conners, n.d.) This is an example that not all natural phenomena can be represented by even and perfect numerals, and sometimes adjustments must be made for our systems to be in sync with the natural world.

Seasons

Geometry Applications

Geometry is a key methodology and subject in studying the Earth in relation to the Sun to determine the passage of time, but the similar geometry of the Earth's movement and distance can be applied in studying the cause of weather and climate. As discussed in the History and Sundials section, there are changing shadows throughout the day that can be used to measure the passage of the day. Likewise, seasonal changes were observed throughout the year to mark the start and end of each annual cycle. In early human history, meteorological seasons were observed to capture the repetition of climate, for people to know when crops would grow more and when shelter was needed in preparation for the colder periods of the year. (Meteorological Versus Astronomical Seasons, n.d.) With the development of geometry and astronomy, astronomical seasons more accurately divided the year into periodical seasons based on the angle of the Sun's radiation in different regions during different times of the year. (Meteorological Versus Astronomical Seasons, n.d.)

Axial Tilt as the Cause of Seasons

So, what exactly is the cause of the seasons? Personally, the initial speculation was that the periods of the year with more warmth was when the sun was slightly closer to the Earth, and the period of the year when it was colder was when the sun was a little farther. The Earth's orbit has an eccentricity of 0.02, eccentricity being a scale that measures how close a planet's orbit is to being a perfect circle. (Williams, 2014) After learning that the Earth's orbit was a bit of an oval rather than a perfect circle, it seemed that this held true, but the opposite is true. On its orbit, the earth experiences about 2 million less miles of distance from the Sun during summer than in the winter. The reason for this is that the Earth has a fixed tilt during its entire revolution around the Sun. The Earth has an axial tilt of about 23.5°, and this tilt is the direct cause of the seasons. This tilt has major contributions to the cause of weather in different areas around the globe, and using trigonometric equations explains how this works. (Sen, 2019) The Earth remains with its center axis pointing in the same direction in space throughout its entire orbit. As it revolves around the sun, the amount of heat the Earth receives from the Sun changes throughout the year. As shown in this diagram, the Earth maintains its axial tilt, but there is variation overtime in where the sun's rays are more directly received.

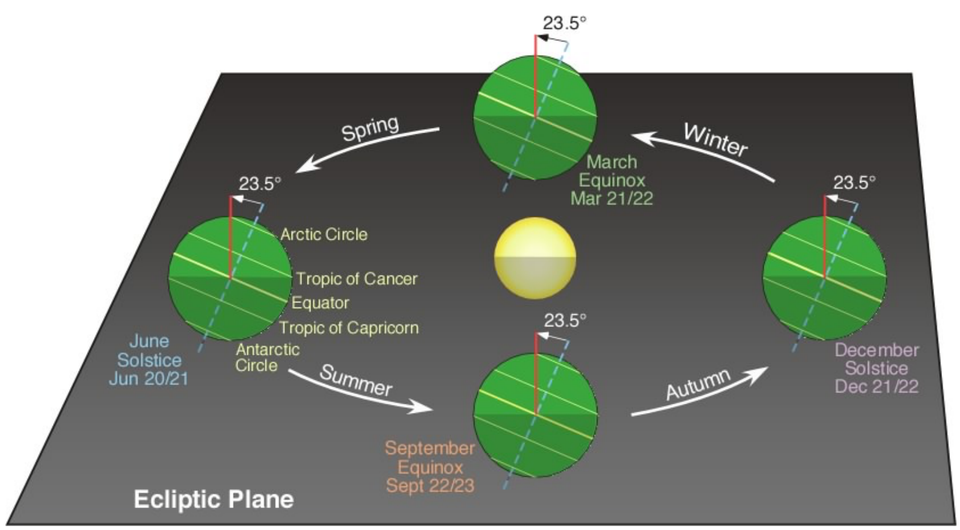

Figure 3 Seasons of the Earth as it orbits the Sun

(Eath-Sun Geometry, n.d.)

Therefore, if the earth was to have a 0° tilt in its axis, seasonal change would be almost nonexistent since the climate and duration of daytime throughout the year would stay consistently the same, with the equator remaining the warmest region of the globe. If this tilt was greater, the seasons would be more intense because it would allow the sun's radiation to be stronger in summer and weaker in winter. (Orbital Variations, 2000) As shown in the picture above, there are two general seasonal types in the year: solstice and equinox. The solstice period is when the sun is positioned at the farthest north or farthest south of the equator, and this is during winter and summer. The equinox period is during fall and spring, when day and night are the same length because the sun hits more evenly on the equator plane. Also, on the globe are the tropic of Cancer and tropic of Capricorn, and these are the exact latitudes that receive the most perpendicular angle of sunlight and heat on summer and winter solstices respectively. (Nilsson)

Seasonal Changes

Specific concepts within solar geometry and physics allow explanation and analysis for the changes in seasons and temperature overtime. A key concept is solar irradiance, which is the amount of light and heat energy on a given surface area region. (Solar Irradiance, 2008) The existence of irradiance, or angled sun rays, is how sundials were able to function in continually allowing us to keep track of the passage of day and seasons. The following diagram displays this concept by showing the geometry of two different sun angles:

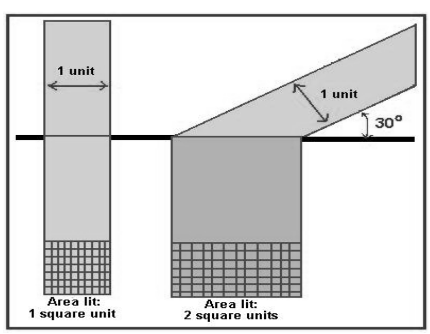

Figure 4 Solar Irradiance of the Sun onto the Earth

(Effect of sun angle on climate, n.d.)

To compare the amount of light energy being received by these different surfaces, the sine of the angles can be taken.

$$\sin(90°)=1$$

$$\sin(30°) = 0.5$$

Though the light does cover more surface area within the 30° sun angle region, it results in a lower irradiation.

The way to track the Earth's current climate based on the angle of sun rays is using the angle of declination. The angle of declination is the angle between the location where the Sun's rays are hitting perpendicularly and the Earth's equator. The value of this angle depends on what day of the year it is. (Simmons, 2018) Using δ for the sun declination angle, and *d* for the days passed since January 1st:

$$\sin\delta=0.39795\times\cos[0.98563\times (d-173)]$$

Using this equation for the date this section was written, August 12th, 2020, the declination angle can be found for that day of the year: the 224th day:

$$\sin\delta=0.39795\times\cos[0.98563\times(224-173)$$

$$\sin\delta=0.39795\times\cos[0.98563\times(51)]$$

$$\sin\delta=0.39795\times\cos[50.26713]$$

$$\sin\delta=0.39795\times0.639202109$$

$$\sin\delta=0.2543732649$$

$$\delta = \sin^{-1} 0.2543732649$$

$$\delta=14.73645134$$

As of August 12th, the Sun is declined to 14.73° compared to the equator. When the declination angle is positive, that is when the Sun's rays are more direct to the northern hemisphere. So right now, it is in between the summer solstice and autumn equinox in the northern hemisphere.

Conclusion

Though the topics of time measurement and the causes of seasons initially seems to be divided topics, they come under a common explanation using geometry applied to astronomy. With a minor involvement of physics, the velocity of the Earth and its distance from other celestial bodies are examined for the sake of keeping track of time and seasons. Going over the history of these examinations is important in understanding ancient mathematical models that support modern, advanced examinations. By analyzing all the geometry from this composition, the repetitive use of sine, cosine, and tangent trigonometry is very notable. Triangles can often be formed to model various angles and lengths between the Earth and the Sun. Spherical geometry is also very valuable because the properties of a sphere allow mathematicians and astronomers to coordinate time zones on the globe.

There is room for limitation, because as a society only so much can determined regarding the universe while remaining on Earth. Modern technology definitely advanced the accuracy of what knowledge society has gained so far, as well as new knowledge in astronomy. As for the evaluated calculations from this research, there were some inaccuracies and unalignments. Even going off of websites that center around science, math, and astronomy, it can be tedious to find all values required to research efficiently. It remains important, though, to use examples to make all of the collected information and facts make sense when conclusively compiled.

Analyzing these pre-existing and already thoroughly understood concepts can be greatly beneficial on daily lives and planning. This is important for weather forecasting, staying on track with any changes in the Earth’s rotation and revolution, and perhaps knowing what time to hold an outdoor event by knowing when it will be warmer or cooler. Additionally, understanding the

Earth's natural climate throughout the year is important in studying the causes of the climate change crisis. (Climate Change: How Do We Know?, n.d.) On the long term scale, society can also determine any changes in the Earth's movement through space by continually studying the geometry and physics in relation to the Sun.

References

8 Different Types of Sundials. (n.d.). Retrieved August 10, 2020, from Home Stratosphere: https://www.homestratosphere.com/types-of-sundials/

au (Astronomical Unit). (n.d.). Retrieved August 30, 2020, from Cneos: https://cneos.jpl.nasa.gov/glossary/au.html

Climate Change: How Do We Know? (n.d.). Retrieved August 30, 2020, from Climate.Nasa: https://climate.nasa.gov/evidence/

Conners, D. (n.d.). *Why do we need leap years?* Retrieved August 21, 2020, from EarthSky: https://earthsky.org/human-world/why-do-we-need-leap-years

Dr. Whitlock, L. A. (n.d.). *Why is there day and night?* Retrieved August 3, 2020, from Star Child: https://starchild.gsfc.nasa.gov/docs/StarChild/questions/question31.html

Finding the AU: How the Transit of Venus Tells Us Our Distance from the Sun. (2012). Retrieved August 27, 2020, from Exploratorium: https://www.exploratorium.edu/venus/question4.html

Finio, B. (2017, September 7). *Measure Earth's Circumference with a Shadow*. Retrieved August 27 2020, from Scientific American: https://www.scientificamerican.com/article/measure-earths-circumference-with-a-shadow/

Finio, B. (2017, September 7). *Measure Earth's Circumfrence with a shadow*. Retrieved August 28, 2020, from Scientific American: https://www.scientificamerican.com/article/measure-earths-circumference-with-a-shadow/

Gedge, D. (2013, July 14). *Understanding Time Zones*. Retrieved August 5, 2020, from Youtube: https://www.youtube.com/watch?v=X1DkiuaFCuA

Heilbron, J. (2020, February 3). *Astronomy and trigonometry*. Retrieved 6 2020, August, from Britannica: https://www.britannica.com/science/geometry/Astronomy-and-trigonometry

Herman, R. (1998, 26 October). *How fast in the earth moving?* Retrieved August 16, 2020, from Scientific American: How fast is the earth moving? - Scientific Americanwww.scientificamerican.com › article › how-fast-is-the-earth-mov

How do sundials work? (2008, November 22). Retrieved August 10, 2020, from Yale Scientific: http://www.yalescientific.org/2008/11/how-do-sundials-work/

How the Four Seasons Came to Be in Greek Mythology. (n.d.). Retrieved August 3, 2020, from Greek Boston: https://www.greekboston.com/culture/mythology/four-seasons/

Howell, E. (2018, June 23). *How fast is earth moving?* Retrieved August 21, 2020, from Space.com: https://www.space.com/33527-how-fast-is-earth-moving.html

Latitude and Longitude. (2019, March 1). Retrieved August 12, 2020, from Open Learn: https://www.open.edu/openlearn/history-the-arts/history/history-science-technology-and-medicine/history-science/latitude-and-longitude

Lucas, J. (2018, December 12). *What is Parallax?* Retrieved August 12, 2020, from Space.com: https://www.space.com/30417-parallax.html

Marshall, J. (2015, June 19). *Why does a circle have 360 degrees?* Retrieved August 18, 2020, from Quick and Dirty Tips: https://www.quickanddirtytips.com/education/math/why-does-a-circle-have-360-degrees?page=1

Meteorological Versus Astronomical Seasons. (n.d.). Retrieved August 21, 2020, from National Centers for Environmental Information: https://www.ncei.noaa.gov/news/meteorological-versus-astronomical-seasons

Nilsson, D. (n.d.). *Solar Geometry.* Retrieved August 24, 2020, from Mypages: http://mypages.iit.edu/~maslanka/SolarGeo.pdf

Orbital Variations. (2000, March 24). Retrieved August 24, 2020, from EarthObservatory.Nasa: https://earthobservatory.nasa.gov/features/Milankovitch/milankovitch_2.php

PV Education. (2019). Retrieved August 26, 2020, from Elevation Angle: https://www.pveducation.org/pvcdrom/properties-of-sunlight/elevation-angle

Rehnberg, M. (2015, January 5). *How did we find the distance to the sun?* Retrieved August 27, 2020, from Phys.org: https://phys.org/news/2015-01-distance-sun.html

Sen, A. (2019, September 10). *Geometry in Seasons | Atmospheric | Celestial | Earth Science | Physics* |. Retrieved August 5, 2020, from Youtube: https://www.youtube.com/watch?v=U5QLaKC_aEk&t=3s

Sharp, T. (2017, October 19). *How Far Earth from the Sun?* Retrieved October 19, 2017, from Space.com: https://www.space.com/17081-how-far-is-earth-from-the-sun.html

Simmons, M. (2018, March 13). *How to calculate the sun's declination*. Retrieved August 27, 2020, from Sciencing: https://sciencing.com/calculate-suns-declination-6904335.html

Solar Irradiance. (2008, Janruary 1). Retrieved August 22, 2020, from Nasa: https://www.nasa.gov/mission_pages/sdo/science/Solar%20Irradiance.html

Toothman, J. (n.d.). *How did ancient civilizations use sundials to tell time?* Retrieved August 8, 2020, from How stuff works: https://electronics.howstuffworks.com/gadgets/clocks-watches/ancient-civilizations-use-sundials.htm

Tourmen. (2020). *Parallax*. Retrieved August 12, 2020, from Wikipedia: https://www.space.com/30417-parallax.html

Vincent, J. (n.d.). *The mathematics of sundials.* Retrieved August 9, 2020, from Files: https://files.eric.ed.gov/fulltext/EJ802706.pdf

Why are there 24 hours in a day? (2011, November 15). Retrieved August 14, 2020, from ABC Science: https://www.abc.net.au/science/articles/2011/11/15/3364432.htm

Williams, M. (2014, November 24). *Earth's orbit around the sun*. Retrieved August 18, 2020, from Phys.org: https://phys.org/news/2014-11-earth-orbit-sun.html